

Déterminer les dérivées des fonctions suivantes :

1. $f(x) = \frac{2x+3}{3x^2-5x+7}$

2. $f(x) = \frac{2x^2+4x-1}{2-7x}$

3. $f(x) = (3x^4 - 5x + 1)^2$

4. $f(x) = \frac{1}{x^2 - 7x + 9}$

5. $f(x) = \frac{2\sqrt{x}}{3-5x}$

6. $f(x) = 4x - \frac{2-5x^2}{x^2+3x+2}$

CORRIGE – NOTRE DAME DE LA MERCI – MONTPELLIER

$$1. f(x) = \frac{2x+3}{3x^2-5x+7}$$

On pose : $u(x) = 2x+3$ et $v(x) = 3x^2-5x+7$

Donc $u'(x) = 2$ et $v'(x) = 6x-5$

$$\begin{aligned} f'(x) &= \frac{2(3x^2-5x+7) - (2x+3) \times (6x-5)}{(3x^2-5x+7)^2} \\ &= \frac{6x^2-10x+14 - (12x^2-10x+18x-15)}{(3x^2-5x+7)^2} \\ &= \frac{6x^2-10x+14-12x^2+10x-18x+15}{(3x^2-5x+7)^2} \\ &= \frac{-6x^2-18x+29}{(3x^2-5x+7)^2} \end{aligned}$$

$$2. f(x) = \frac{2x^2+4x-1}{2-7x}$$

On pose : $u(x) = 2x^2+4x-1$ et $v(x) = 2-7x$

Donc $u'(x) = 4x+4$ et $v'(x) = -7$

$$\begin{aligned} f'(x) &= \frac{(4x+4)(2-7x) - (2x^2+4x-1) \times (-7)}{(2x^2+4x-1)^2} \\ &= \frac{(8x-28x^2+8-28x) - (-14x^2-28x+7)}{(2x^2+4x-1)^2} \\ &= \frac{8x-28x^2+8-28x+14x^2+28x-7}{(2x^2+4x-1)^2} \\ &= \frac{-14x^2+8x+1}{(2x^2+4x-1)^2} \end{aligned}$$

$$3. f(x) = (3x^4-5x+1)^2$$

On pose : $u(x) = 3x^4-5x+1$

Ainsi : $u'(x) = 3 \times 4x^3 - 5 = 12x^3 - 5$

$$f'(x) = 2(3x^4-5x+1)(12x^3-5)$$

$$4. f(x) = \frac{1}{x^2-7x+9}$$

La dérivée de $\left(\frac{1}{v}\right)$ est $\frac{-v'}{v^2}$. Autre méthode :

On pose : $u(x) = 1$ et $v(x) = x^2-7x+9$

Donc $u'(x) = 0$ et $v'(x) = 2x-7$

$$f'(x) = \frac{0 \times (x^2-7x+9) - 1 \times (2x-7)}{(x^2-7x+9)^2} = \frac{-2x+7}{(x^2-7x+9)^2}$$

$$5. f(x) = \frac{2\sqrt{x}}{3-5x}$$

On pose : $u(x) = 2\sqrt{x}$ et $v(x) = 3-5x$

Donc $u'(x) = 2 \times \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$ et $v'(x) = -5$

$$\begin{aligned} f'(x) &= \frac{\frac{1}{\sqrt{x}}(3-5x) - 2\sqrt{x} \times (-5)}{(3-5x)^2} \\ &= \frac{\frac{1}{\sqrt{x}}(3-5x) + 10\sqrt{x}}{(3-5x)^2} = \frac{\frac{1}{\sqrt{x}}(3-5x) + 10\sqrt{x} \times \frac{\sqrt{x}}{\sqrt{x}}}{(3-5x)^2} \\ &= \frac{\frac{1}{\sqrt{x}}(3-5x) + \frac{10x}{\sqrt{x}}}{(3-5x)^2} = \frac{\frac{3-5x+10x}{\sqrt{x}}}{(3-5x)^2} = \frac{5x+3}{\sqrt{x} \times (3-5x)^2} \end{aligned}$$

$$6. f(x) = 4x - \frac{2-5x^2}{x^2+3x+2}$$

On pose : $u(x) = 2-5x^2$ et $v(x) = x^2+3x+2$

Donc $u'(x) = -10x$ et $v'(x) = 2x+3$

$$\begin{aligned} f'(x) &= 4 - \frac{(-10x)(x^2+3x+2) - (2-5x^2)(2x+3)}{(x^2+3x+2)^2} \\ &= 4 - \frac{(-10x^3-30x^2-20x) - (4x+6-10x^3-15x^2)}{(x^2+3x+2)^2} \\ &= 4 - \frac{-10x^3-30x^2-20x-4x-6+10x^3+15x^2}{(x^2+3x+2)^2} \\ &= 4 - \frac{-15x^2-24x-6}{(x^2+3x+2)^2} \end{aligned}$$

